Basic Concepts

Objective:
To define the basic concepts of circuit analysis.

Circuit:
It is an interconnection of electrical elements in a closed path by conductors (wires).

Basic Circuit Definitions

Node: Any junction point between two or more elements (1,2,3 and 4)

Essential node: A Junction between MORE than two elements (2 and 4)

Non-essential node: A Junction between only two elements (1 and 3)

Branch: The part of the circuit that lies between two nodes (A,B,C,D and E)

Loop: a closed path in a circuit (loop1, loop2 and loop3)

Mesh: a loop that does not contain any loops (Mesh1 and Mesh2)

Current: Current measures the flow of positive charge through wires and elements in a circuit.

\[ i(t) = \frac{dq}{dt} \]

Unit: \( \frac{Coulomb}{second} \) = Ampere (A)

The total charge:
\[ q(t) = q(t_0) + \int_{t_0}^{t} i(t) \, dt \]

- \( t_0 \): Initial time
- \( t \): Final time
- \( q(t_0) \): Initial charge

**Current Reference Directions**
The current represents by an arrow to indicate its direction.

When we analyze the circuit the direction of the current is selected arbitrary. After solving for the current values, if the current is negative this means the direction of the current should be opposite to our initial choice. For example, in the circuit below if after solving we find that \( i_1 \) equal to \(-2 \text{ A}\), than the direction of \( i_1 \) is opposite to our initial choice.

**Double-subscript Notation for Currents:**

\[ l_{ab} = -l_{ba} \]

**Type of Currents:**

- **Direct Current (DC):** the current is constant with time.
  
  \[ 5 \text{ A} \]

- **Alternating Current (AC):** The current varies with time: periodically reverses direction.
Voltage:
The voltage associated with a circuit element is the energy transferred per unit of charge.

\[ v(t) = \frac{dw}{dq} \]

Unit: Volt (V)

Voltages can be constant with time or vary. Constant voltages are called DC voltages. On the other hand, voltages that change in magnitude and alternate polarity with time are called ac voltages.

Double-subscript Notation for Voltages:

\[ v_{ab} = -v_{ba} \]

The voltage \( v_{ab} \) has a reference polarity that is positive at point \( a \) and negative at \( b \). The voltage \( v_{ba} \) has a reference polarity that is positive at point \( b \) and negative at \( a \). Another way to indicate a voltage and its reference is to use an arrow. The positive reference corresponds to the head of the arrow.

Power
Power is the rate of energy transfer.

\[ p = \frac{dw}{dt} \]

In circuit analysis the power is described in term of voltage and current:

\[ p = \frac{dw}{dt} = \frac{dv}{dq} \cdot \frac{dq}{dt} = V \cdot I \]

Unit: \( \frac{Joule}{second} = Watt \ (W) \)
**Passive Reference Configuration:**
In the following figure, notice that the current reference enters the positive polarity of the voltage. This arrangement is called passive reference configuration.

![Passive Reference Configuration Diagram]

**Note:**
If the current reference enters the negative end of the reference polarity we compute the power as:

\[ P = -V \cdot I \]

A positive value for \( p \) indicates the energy is absorbed by the element and a negative value of \( p \) shows that the energy is supplied by the element.

Power is conserved over all any circuit. This means that the magnitude of the supplied power MUST be equal to the magnitude of the absorbed power (see the last example of this note).

**Example:**
Calculate the power for each element.

![Element Diagrams]

**Solution**
- \( P_a = v_a \cdot i_a = 12 \times 2 = 24 \text{ w} \) (Absorbed)
- \( P_b = -v_b \cdot i_b = -12 \times 1 = -12 \text{ w} \) (Supplied)
- \( P_c = v_c \cdot i_c = 4 \times (-2) = -8 \text{ w} \) (Supplied)
Energy
To calculate the energy \( W \) delivered to a circuit between time instants \( t_1 \) and \( t_2 \), we integrate the power:

\[
W = \int_{t_1}^{t_2} p(t) \, dt
\]

Electrical Elements:

1. Resistor
A resistor is a two-terminal electronic component that produces a voltage across its terminals that is proportional to the electric current passing through it in accordance with Ohm's law.

\[
R = \frac{V}{I} \quad \text{(Ohm’s Law)}
\]

Unit: Ohm (Ω)

Conductance (G)
Another important quantity in circuit analysis is known as conductance (G)

\[
G = \frac{1}{R}
\]

Unit: mho (\( \frac{1}{Ω} \)) or Siemens (S)
Using the conductance:

\[
R = \frac{V}{I}
\]

\[
G = \frac{I}{V}
\]

\[
l = G \cdot V
\]
2. **Independent Voltage Source**
   An ideal Independent voltage source maintains a specified voltage across its terminal. The voltage across an independent voltage source is independent of the elements connected to it and of the current flowing through it.

![Diagram of independent voltage source](image1)

3. **Independent Current Source**
   An ideal Independent current source forces a specified current to flow through itself. The current through an independent current source is independent of the elements connected to it and of the voltage across it.

![Diagram of independent current source](image2)
Basic Circuit Analysis Laws

We analyze electrical circuits in order to find the voltage, current and power of an electrical element. In circuit analysis there are three basic laws that are commonly used to analyze circuits.

1. **Ohm's Law:**
The current through a resistor is directly proportional to the potential difference (voltage) across the resistor, and inversely proportional to the resistor:

   \[ I = \frac{V}{R} \]

   \[ R = \frac{V}{I} \]

2. **Kirchhoff’s Current Law (KCL)**

   The algebraic sum of the currents leaving any node is zero.

   For the above figure:

   - Apply KCL at node a: \(-i_1 + i_2 + i_3 = 0\)
   - Apply KCL at node b: \(-i_4 - i_5 + i_6 = 0\)
   - Apply KCL at node b: \(-i_7 + i_8 = 0 \rightarrow i_7 = i_8\)

**Series Connection**

If two elements are connected in series as shown below, then they share the same current.
**Example:**
Find the value of the $i_1$ and $i_2$:

![Circuit Diagram]

**Solution:**

In the above figure, each oval shape indicates to a common node.

Apply KCL at node $a$

\[ i_1 - 7 + 2 = 0 \]
\[ i_1 = 5 \text{ A} \]

Apply KCL at node $b$

\[ -i_1 + i_2 - 7 = 0 \]
\[ i_2 = i_1 + 7 = 5 + 7 = 12 \text{ A} \]
3. **Kirchhoff’s Voltage Law (KVL)**
   The algebraic sum of the voltages around any loop is zero.

   \[ -V_1 + V_2 + V_3 - V_4 = 0 \]

**Parallel Connection**
When two elements are connected together at each node (as shown below), they form a parallel connection and the voltage is the same across both elements:

To prove that, apply KVL:

\[ -V_a + V_b = 0 \]

\[ V_a = V_b \]

**Note**: in the following, each two resistors are connected in parallel and the four connections are equivalent to each other:
**Example:**
Apply the KVL in all the loops in the following circuit

![Circuit Diagram](image)

**Solution:**
The circuit has three loops. We can assume the direction of each loop either in clock-wise or anti-clock-wise direction.

![Circuit with Loop Indication](image)

Apply KVL at loop #1

\[-V_s + V_1 + V_2 - V_5 = 0\]

Apply KVL at loop #1

\[-V_2 + V_3 + V_4 = 0\]

Apply KVL at loop #1

\[-V_s + V_1 + V_3 + V_4 - V_5 = 0\]

**Example:**
By using KVL find the value of the $V_1$
Solution:

Apply KVL around the loop:

\[-10 + V_1 + 8 = 0 \Rightarrow V_1 = 2 \text{ v}\]

Example

Find the voltage across the 200 Ω resistor.

Solution

The polarities of the voltages across the resistors are not given, so we assume the polarity across each resistor as show below (Note: you can assume the opposite polarity).

Apply KVL around the loop:

\[-10 + V_1 + 40 + V_2 = 0 \quad (1)\]

The above equation has two unknown values, so we cannot solve for the unknown values by using only one equation. However, we can assume the direction of the current (i) as show below, and apply the ohm's law for the voltages in equation (1) to find the comment current (i).
Example:
In the following circuit find $V_1$ and $V_2$.

Solution
Since the 4Ω and 1Ω resistors are connected in parallel with the voltage source (5 V), then they share the same voltage:

$V_1 = V_2 = 5 \text{ V}$

Open and Short Circuit Concept

- In case of short circuit, $R=0$ and the voltage across it is equal to zero ($V=0$)
- In case of open circuit, $R=\infty$ and the current is equal to zero ($I=0$)
In order to find $R$, we should find $V$. For the following circuit:

Example (Extra class problem):
For the following circuit:

a. Find $V_x$ and $R$.

b. Prove that the supplied power is equal to the absorbed power.

Solution:
(a)
The current passing through the 1Ω is equal to 7 A, because the 1Ω resistor and the current supply are connected in series.

Note: In a resistor the current enters the positive polarity of the voltage across it.

In order to find $R$, we should find $V_R$ and $I_R$; then
To find $I_R$: apply KCL at node a:

$$R = \frac{V_R}{I_R}$$

To find $V_R$: apply KVL around loop#1:

$$-7 + I_R + 3 = 0$$

$$I_R = 4 \text{A}$$

Apply Ohm’s law:

$$-V_R + 3x2 + 3x4 = 0$$

$$V_R = 18 \text{v}$$

$$R = \frac{V_R}{I_R} = \frac{18}{4} = 4.5 \text{Ω}$$

(b) To find the voltage ($V_x$) across the current source, apply the KVL around the loop #2:

$$-V_x + V_1 \Omega + V_2 \Omega + V_4 \Omega = 0$$

Apply Ohm’s law $V=IR$:

$$-V_x + 7x1 + 3x2 + 3x4 = 0$$

$$V_x = 25 \text{v}$$

$$P = VI$$

$$P_{current \ supply} = -25 \times 7 = -175 \text{ W}$$ (Supplied, because it has a negative sign). The current source supplies 175 W.

The power absorbed by each resistor can be calculated as:

$$P_{1 \Omega} = 7^2 \times 1 = 49 \text{ W}$$

$$P_{4.5 \Omega} = 4^2 \times 4.5 = 72 \text{ W}$$

$$P_{2 \Omega} = 3^2 \times 2 = 18 \text{ W}$$

$$P_{4 \Omega} = 3^2 \times 4 = 36 \text{ W}$$

The summation of the magnitude of the absorbed power is equal to the summation of the magnitude of the supplied power.

$$P_{sum \ of \ the \ absorbed \ power} = P_{1 \Omega} + P_{4.5 \Omega} + P_{2 \Omega} + P_{4 \Omega} = 49 + 72 + 18 + 36 = 175 \text{ W}$$

$$P_{sum \ of \ the \ supplied \ power} = -175 \text{ W}$$

$$|P_{sum \ of \ the \ supplied \ power}| = |P_{sum \ of \ the \ absorbed \ power}|$$